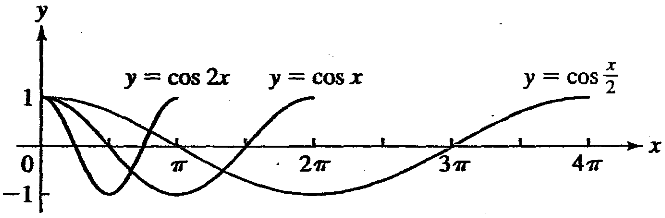
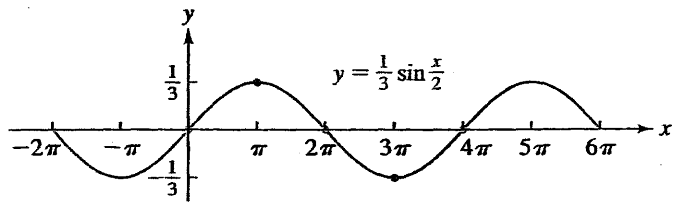
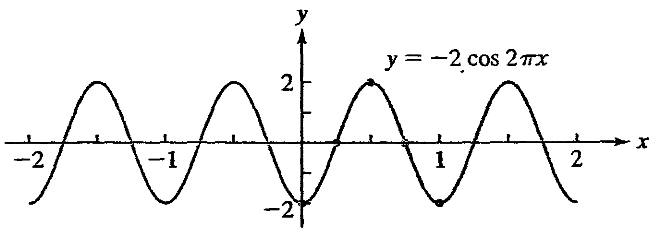
2.



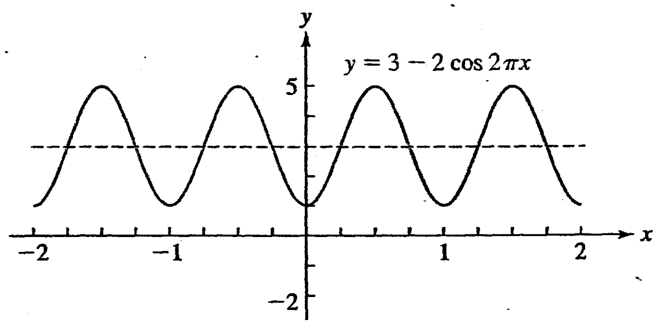

3. Amplitude = 1/3; Period = 4π




4. Amplitude = 2; Period = 1




5.




6. $y = -0.065 \cos 524\pi t$ 7. $y = 0.4 \sin 4\pi t$

## EXERCISE 3.2

**A** *Make a sketch of each trigonometric function without looking at the text or using a calculator. Label each point where the graph crosses the x axis.*

1. $y = \sin x, \quad -2\pi \le x \le 2\pi$
2. $y = \cos x, \quad -2\pi \le x \le 2\pi$

*State the amplitude and period for each equation, and gra it over the indicated interval.*

3. $y = -2 \sin x, \quad 0 \le x \le 4\pi$
4. $y = -3 \cos x, \quad 0 \le x \le 4\pi$
5. $y = \frac{1}{2} \sin x, \quad 0 \le x \le 2\pi$

6. $y = \frac{1}{3}\cos x, \quad 0 \le x \le 2\pi$
7. $y = \sin 2\pi x, \quad -2 \le x \le 2$
8. $y = \cos 4\pi x, \quad -1 \le x \le 1$
9. $y = \cos \dfrac{x}{4}, \quad 0 \le x \le 8\pi$
10. $y = \sin \dfrac{x}{2}, \quad 0 \le x \le 4\pi$

3 *In Problems 11–16, state the amplitude and period for each equation, and graph it over the indicated interval.*

11. $y = 2 \sin 4x, \quad -\pi \le x \le \pi$
12. $y = 3 \cos 2x, \quad -\pi \le x \le \pi$
13. $y = \frac{1}{3}\cos 2\pi x, \quad -2 \le x \le 2$
14. $y = \frac{1}{2}\sin 2\pi x, \quad -2 \le x \le 2$
15. $y = -\dfrac{1}{4}\sin\dfrac{x}{2}, \quad -4\pi \le x \le 4\pi$
16. $y = -3\cos\dfrac{x}{2}, \quad -4\pi \le x \le 4\pi$

*Problems 17 and 18 involve oscillating objects. For each problem, find an equation of the form $y = A \sin Bt$ or $y = A \cos Bt$ that satisfies the given conditions ($y$ is displacement from a central position at time $t$).*

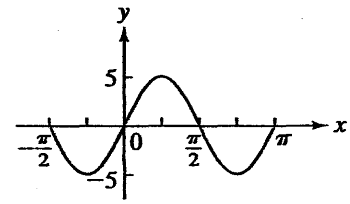
17. Displacement is 0 ft when $t$ is 0, amplitude is 2 ft, and period is 2 sec.
18. Displacement from the $t$ axis is 5 cm when $t$ is 0, amplitude is 5 cm, and period is 0.1 sec.
19. Describe what happens to the size of the period of $y = A \sin Bx$ as $B$ increases without bound.
20. Describe what happens to the size of the period of $y = A \cos Bx$ as $B$ decreases through positive values toward 0.

*Graph each equation over the indicated interval.*

21. $y = -1 + \frac{1}{3}\cos 2\pi x, \quad -2 \le x \le 2$
22. $y = -\frac{1}{2} + \frac{1}{2}\sin 2\pi x, \quad -2 \le x \le 2$
23. $y = 2 - \dfrac{1}{4}\sin\dfrac{x}{2}, \quad -4\pi \le x \le 4\pi$
24. $y = 3 - 3\cos\dfrac{x}{2}, \quad -4\pi \le x \le 4\pi$

*In Problems 25–28, find the equation of the form $y = A \sin Bx$ that produces the indicated graph.*

25.

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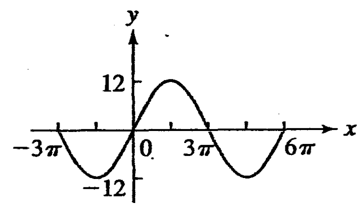
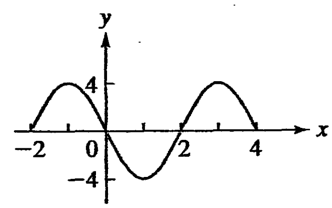



27.

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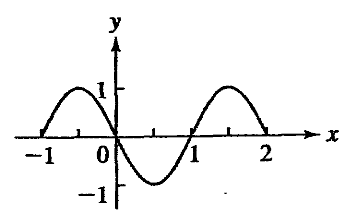
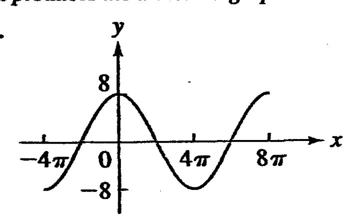



*In Problems 29–32, find the equation of the form $y = A \cos Bx$ that produces the indicated graph.*

29.



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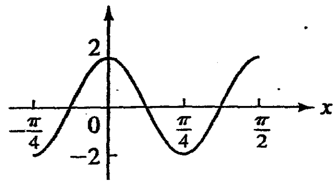


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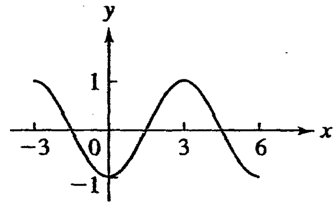
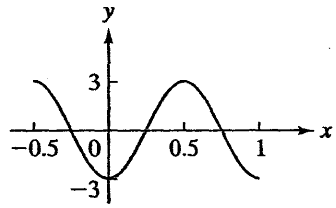



32.




*In Problems 33–38, graph the given equation on a graphing utility. (Adjust the ranges in the viewing windows so that you see at least two periods of a particular function.) Find an equation of the form $y = k + A \sin Bx$ or $y = k + A \cos Bx$ that has the same graph. These problems suggest the existence of further identities in addition to the basic identities discussed in Section 2.6. Further identities are discussed in detail in Chapter 4.*

**33.** $y = \sin x \cos x$

**34.** $y = \cos^2 x - \sin^2 x$

**35.** $y = 2\cos^2 x$

**36.** $y = 2\sin^2 x$

**37.** $y = 2 - 4\sin^2 2x$

**38.** $y = 6\cos^2 \frac{x}{2} - 3$

*Problems 39 and 40 graphically explore the relationships of the graphs of $y = \sin(x + C)$ and $y = \cos(x + C)$ relative to the corresponding graphs of $y = \sin x$ and $y = \cos x$. This topic is discussed in detail in the next section.*

**39.** (A) Graph $y = \sin(x + C)$, $-2\pi \le x \le 2\pi$, for $C = 0$ and $C = -\pi/2$ in one viewing window, and for $C = 0$ and $C = \pi/2$ in another viewing window. (Experiment with other positive and negative values of $C$.)

(B) Based on the graphs in part (A), describe how the graph of $y = \sin(x + C)$ is related to the graph of $y = \sin x$ for various values of $C$.

**40.** (A) Graph $y = \cos(x + C)$, $-2\pi \le x \le 2\pi$, for $C = 0$ and $C = -\pi/2$ in one viewing window, and for $C = 0$ and $C = \pi/2$ in another viewing window. (Experiment with other positive and negative values of $C$.)

(B) Based on the graphs in part (A), describe how the graph of $y = \cos(x + C)$ is related to the graph of $y = \cos x$ for various values of $C$.

C **41.** From the graph of $f(x) = \cos^2 x$, $-2\pi \le x \le 2\pi$, on a graphing utility, determine the period of $f$; that is, find the smallest positive number $p$ such that $f(x + p) = f(x)$.

**42.** From the graph of $f(x) = \sin^2 x$, $-2\pi \le x \le 2\pi$, on a graphing utility, determine the period of $f$; that is, find the smallest positive number $p$ such that $f(x + p) = f(x)$.

**43.** The table in the figure was produced using a table feature for a particular graphing calculator. Find a function of the form $y = A \sin Bx$ or $y = A \cos Bx$ that will produce the table.

| X | Y1 |
|---|---|
| 0.0 | 2.0 |
| .5 | 1.0 |
| 1.0 | -1.0 |
| 1.5 | -2.0 |
| 2.0 | -1.0 |
| 2.5 | 1.0 |
| 3.0 | 2.0 |

X=0

**44.** The table in the figure was produced using a table feature for a particular graphing calculator. Find a function of the form $y = A \sin Bx$ or $y = A \cos Bx$ that will produce the table.

| X | Y1 |
|---|---|
| 0.0 | 0.0 |
| 1.0 | -3.0 |
| 2.0 | 0.0 |
| 3.0 | 3.0 |
| 4.0 | 0.0 |
| 5.0 | -3.0 |
| 6.0 | 0.0 |

X=0

## Applications

*In these applications, assume all given values are exact unless indicated otherwise.*

**45. Electrical Circuits** The voltage $E$ in an electrical circuit is given by $E = 110 \sin 120\pi t$, where $t$ is time in seconds. What are the amplitude and period of the function? What is the frequency of the function? Graph the function for $0 \le t \le \frac{3}{60}$.

46. **Spring–Mass System** The equation $y = -4 \cos 8t$, where $t$ is time in seconds, represents the motion of a weight hanging on a spring after it has been pulled 4 cm below its equilibrium point and released (see the figure). What are the amplitude, period, and frequency of the function? [Air resistance and friction (damping forces) are neglected.] Graph the function for $0 \le t \le 3\pi/4$.

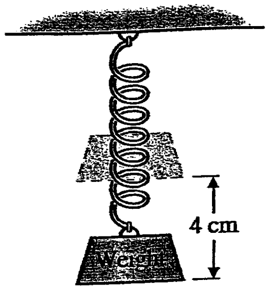



Figure for 46

* 47. **Electrical Circuits** If the voltage $E$ in an electrical circuit has an amplitude of 12 V and a frequency of 40 Hz, and if $E = 12$ V when $t = 0$ sec, find an equation of the form $E = A \cos Bt$ that gives the voltage at any time $t$.

* 48. **Spring–Mass System** If the motion of the weight in Problem 46 has an amplitude of 6 in. and a frequency of 2 Hz, and if its position when $t = 0$ sec is 6 in. above its position at rest (above the rest position is positive and below is negative), find an equation of the form $y = A \cos Bt$ that describes the motion at any time $t$. (Neglect any damping forces—that is, air resistance and friction.)

* 49. **Floating Objects**
(A) A 3 m × 3 m × 1 m float in the shape of a rectangular solid is observed to bob up and down with a period of 1 sec. What is the mass of the float (in kilograms, to three significant digits)?
(B) Write an equation of motion for the float in part (A) in the form $y = D \sin Bt$, assuming the amplitude of the motion is 0.2 m.
(C) Graph the equation found in part (B) for $0 \le t \le 2$.

* 50. **Floating Objects**
(A) A cylindrical buoy with diameter 0.6 m is observed (after being pushed) to bob up and down with a period of 0.4 sec. What is the mass of the buoy (to the nearest kilogram)?
(B) Write an equation of motion for the buoy in the form $y = D \sin Bt$, assuming the amplitude of the motion is 0.1 m.
(C) Graph the equation for $0 \le t \le 1.2$.

51. **Physiology** A normal seated adult breathes in and exhales about 0.80 liter of air every 4.00 sec. The volume of air $V(t)$ in the lungs (in liters) $t$ seconds after exhaling is modeled approximately by

$$V(t) = 0.45 - 0.40 \cos \frac{\pi t}{2} \qquad 0 \le t \le 8$$

(A) What is the maximum amount of air in the lungs, and what is the minimum amount of air in the lungs? Explain how you arrived at these numbers.
(B) What is the period of breathing?
(C) How many breaths are taken per minute? Show your computation.
(D) Graph the equation on a graphing utility for $0 \le t \le 8$ and $0 \le V \le 1$. Find the maximum and minimum volumes of air in the lungs from the graph. [Compare with part (A).]

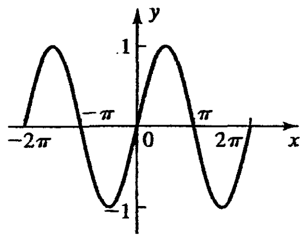
52. **Physiology** When you have your blood pressure measured during a physical examination, the nurse or doctor will record two numbers as a ratio—for example, 120/80. The first number (systolic pressure) represents the amount of pressure in the blood vessels when the heart contracts (beats) and pushes blood through the circulatory system. The second number (diastolic pressure) represents the pressure in the blood vessels at the lowest point between the heartbeats, when the heart is at rest. According to the National Institutes of Health, normal blood pressure is below 130/85, moderately high is from 160/100 to 179/109, and severe is higher than 180/110. The blood pressure of a particular person with moderately high blood pressure is modeled approximately by

$$P = 135 + 30 \cos 2.5\pi t \qquad t \ge 0$$

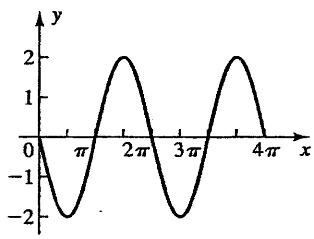
where $P$ is pressure in millimeters of mercury $t$ seconds after the heart contracts.
(A) Express the person's blood pressure as an appropriate ratio of two numbers. Explain how you arrived at these numbers.
(B) What is the period of the heartbeat?
(C) What is the pulse rate in beats per minute? Show your computation.
(D) Graph the equation in a graphing utility for $0 \le t \le 4$, $100 \le P \le 170$. Explain how you would find the person's blood pressure as a ratio using the graph. Find the person's blood pressure using this method. [Compare with part (A).]

## Exercise 3.2

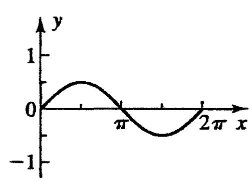
1.



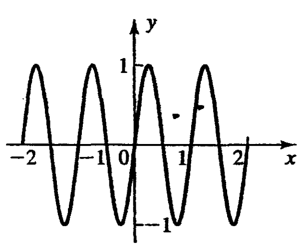
3. Amplitude = 2; Period = $2\pi$



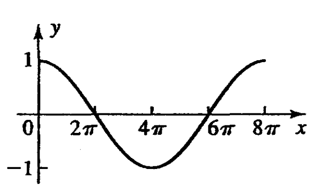
5. Amplitude = $\frac{1}{2}$; Period = $2\pi$



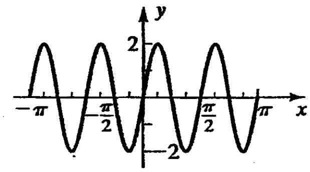
7. Amplitude = 1; Period = 1



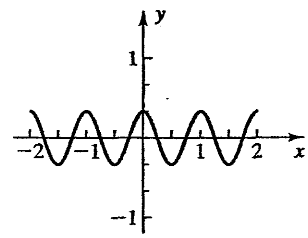
9. Amplitude = 1; Period = $8\pi$



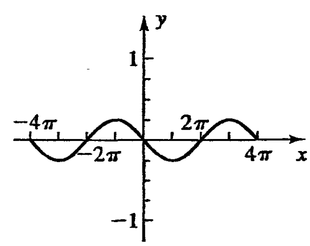
11. Amplitude = 2; Period = $\pi/2$



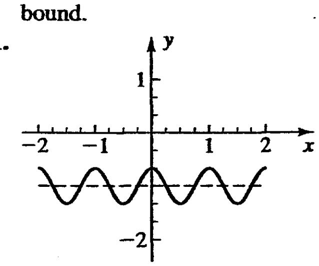
13. Amplitude = $\frac{1}{3}$; Period = 1



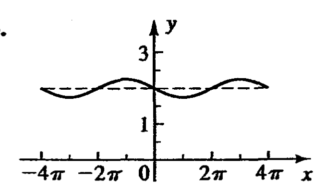
15. Amplitude = $\frac{1}{4}$; Period = $4\pi$



17. $y = 2 \sin \pi t$ or $y = -2 \sin \pi t$
19. Since $P = 2\pi/B$, $P$ approaches 0 as $B$ increases without bound.
21.



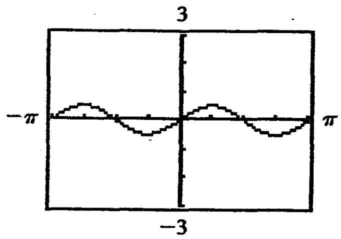
23.



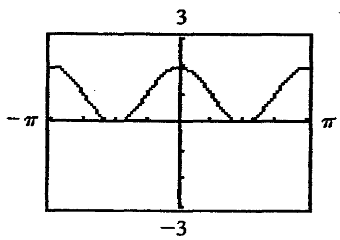
25. $y = 5 \sin 2x$ 27. $y = -4 \sin(\pi x/2)$

**29.** $y = 8\cos(x/4)$ **31.** $y = -\cos(\pi x/3)$

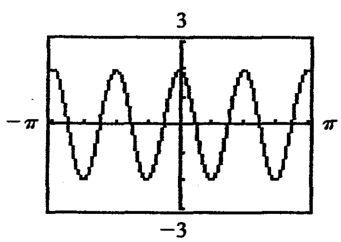
**33.** $y = 0.5\sin 2x$



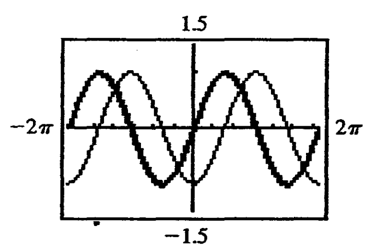

**35.** $y = 1 + \cos 2x$



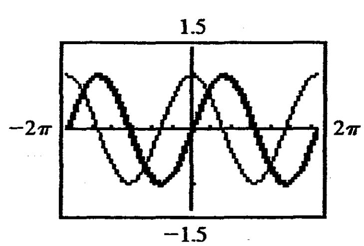

**37.** $y = 2\cos 4x$




**39.** (A) $C = 0$ and $-\pi/2$:




$C = 0$ and $\pi/2$:

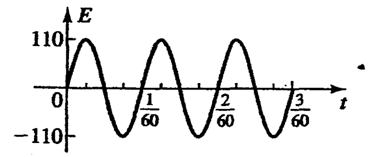



(B) If $C < 0$, then the graph of $y = \sin x$ is shifted $|C|$ units to the right. If $C > 0$, then the graph of $y = \sin x$ is shifted $C$ units to the left.

**41.** $p = \pi$

**43.** $y = 2\cos\dfrac{2\pi x}{3}$

**45.** Amplitude = 110; Period = $\frac{1}{60}$ sec; Frequency = 60 Hz

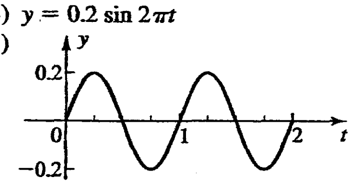



**47.** $E = 12\cos 80\pi t$

**49.** (A) 2,220 kg

(B) $y = 0.2\sin 2\pi t$

(C)



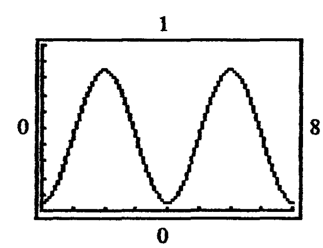

**51.** (A) Max vol = 0.85 liter; Min vol = 0.05 liter; $0.40\cos\dfrac{\pi t}{2}$ is maximum when $\cos\dfrac{\pi t}{2}$ is 1 and is minimum when $\cos\dfrac{\pi t}{2}$ is $-1$. Therefore,

max vol = 0.45 + 0.40 = 0.85 liter and

min vol = 0.45 − 0.40 = 0.05 liter.

(B) 4.00 sec (C) 60/4 = 15 breaths/min

(D) Max vol = 0.85 liter; Min vol = 0.05 liter




**53.**